Mon. Not. R. Astron. Soc. **000**, 1–11 (2003) Printed 2 February 2008 (MN LaTeX style file v2.2)

Recombination Induced Softening and Reheating of the Cosmic Plasma

P. K. Leung[1,2]⋆, C. W. Chan[1,3]†, M. -C. Chu[1]‡
[1] *Physics Department, The Chinese University of Hong Kong, Hong Kong*
[2] *Department of Astronomy, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA*
[3] *Department of Physics, New York University, NY 10003, USA*

Accepted XXXX XXX XX. Received XXXX XXX XX

ABSTRACT

The atomic recombination process leads to a softening of the matter equation of state as reflected by a reduced generalized adiabatic index, with accompanying heat release. We study the effects of this recombination softening and reheating of the cosmic plasma on the ionization history, visibility function, Cold Dark Matter (CDM) transfer function, and the Cosmic Microwave Background (CMB) spectra. The resulting modification of the CMB spectrum is 1/10 of *WMAP*'s current error and is comparable to *PLANCK*'s error. Therefore, this effect should be considered when data with higher accuracy are analysed.

Key words: Cosmology - cosmic microwave background, theory

1 INTRODUCTION

Recent advances in the measurement of Cosmic Microwave Background (CMB) Anisotropies have attracted considerable interest in the field (for latest CMB observations, please refer to Bennett et al. (2003), website of *WMAP* Satellite[1], website of *PLANCK* Satellite[2], Max Tegmark's CMB experiment website[3]; for recent reviews of CMB theory, please read Hu & Dodelson (2002) and Durrer (2001)). The cosmic microwave photons are believed to be remnants of radiations produced in the hot early universe and carry information about the state of the universe during the recombination epoch, when the photon cross section is drastically reduced due to the formation of neutral atoms. It is therefore possible to put tight constraints on cosmological theories using high precision CMB Anisotropies data, provided that the theoretical computation of the CMB Anisotropies also achieves at least the same level of precision. We need to understand the recombination process well if we want to extract information from CMB Anisotropies correctly.

In the standard calculation of the recombination process (Seager et al. 1999, 2000), the baryons are treated as ideal gas with an adiabatic index of $\gamma = 5/3$. However, as the protons and electrons combine to form neutral atoms, the equation of state (EOS) of matter is softened, as reflected by a reduced generalized adiabatic index (Chandrasekhar 1939; Mihalas & Mihalas 1999), which depends on the ionization fraction. This change in the generalized adiabatic index is most drastic during the recombination, and the heat produced delays the completion of the process. In this paper, we study the effect of this recombination softening and reheating of the cosmic plasma on the ionization history, visibility function, the CMB spectra, and the cold dark matter (CDM) power spectra.

We first present a brief review of the standard recombination calculation in Section 2, which is then followed by a discussion of the modifications of the standard equations to take into account the recombination softening and reheating in Section 3. We then present our numerical results in Section 4, and summarize our discussion in Section 5.

⋆ E-mail: poleung@astro.uiuc.edu
† E-mail: cwc243@nyu.edu
‡ E-mail: mcchu@phy.cuhk.edu.hk
[1] Website of *WMAP* is http://map.gsfc.nasa.gov/
[2] Website of *PLANCK* is http://astro.estec.esa.nl/SA-general/Projects/Planck/
[3] Max Tegmark's CMB experiment website is http://www.hep.upenn.edu/∼max/cmb/experiments.html

2 STANDARD RECOMBINATION CALCULATION

2.1 Expansion of the universe

The recombination process did not happen instantaneously because of finite interaction and transition time. Thus we need to follow the time evolution of the ionization fraction of hydrogen and helium and also the baryon temperature. The rate equations can be written in terms of time t, which is related to the redshift z by

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -(1+z)H(z) \, , \tag{1}$$

where $H(z)$ is the Hubble parameter. The expansion of the universe as a whole is described by the cosmic scale factor $a(t) = 1/(1+z)$.

The evolution of the Hubble parameter is determined by the Friedmann equation

$$H(z)^2 = H_0^2 \left[\Omega_{\mathrm{R}}(1+z)^4 + \Omega_{\mathrm{M}}(1+z)^3 + \Omega_{\mathrm{K}}(1+z)^2 + \Omega_{\Lambda}(1+z)^{3(1+w)}\right] \, , \tag{2}$$

where H_0 is the present value of the Hubble parameter and can be written as $100h\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$. The Ω's are the density parameters, with subscripts R referring to the radiation contribution, M the matter contribution (including both baryon B and cold dark matter C), K the curvature contribution, and Λ the dark energy contribution. w is the EOS index of dark energy, which is assumed to be constant. The radiation density parameter is calculated by

$$\Omega_{\mathrm{R}} = \frac{8\pi G(1+f_\nu)U}{3(H_0 c)^2} \, , \tag{3}$$

where $f_\nu = 3 \times (7/8) \times (4/11)^{4/3} \approx 0.681$ is the contribution of neutrinos to the energy density of photons assuming three massless neutrino types, G is the gravitational constant, and U is the photon energy density given by Stefan's law $U = a_{\mathrm{R}} T_{\mathrm{R}}^4$.

2.2 Ionization fractions

We now consider the equations for the ionization fractions. Due to the higher ionization energy of helium comparing to hydrogen, helium recombined first. A helium atom has two electrons, and so there are two processes of recombination, namely He II recombination (from He^{2+} to He^{+}) and He I recombination (from He^{+} to He atom). This He II recombination process is so fast that it can be approximated by using the Saha equation (Seager et al. 1999, 2000), at least in high-Ω_{B} models. For low-Ω_{B} models, the He II recombination is slightly slower than the Saha recombination. But since the He II recombination occured too early and had negligible effect on the CMB power spectrum, it should be adequate to calculate He II recombination by using the Saha equation.

The Saha equation for He II recombination is[4]

$$\frac{(x_e - 1 - f_{\mathrm{He}})x_e}{1 + 2f_{\mathrm{He}} - x_e} = \frac{(2\pi m_e k T_{\mathrm{M}})^{3/2}}{h^3 n_{\mathrm{H}}} \mathrm{e}^{-\chi_{\mathrm{HeII}}/kT_{\mathrm{M}}} \, , \tag{4}$$

where $x_e = n_e/n_{\mathrm{H}}$ is the ionization fraction with n_e being the electron number density, n_{H} the total number density of hydrogen nuclei, $f_{\mathrm{He}} \equiv n_{\mathrm{He}}/n_{\mathrm{H}} = Y_{\mathrm{p}}/4(1-Y_{\mathrm{p}})$ the ratio of the numbers of helium nuclei to hydrogen nuclei, n_{He} the total number density of helium nuclei, and Y_{p} is the primordial He abundance by mass. Here, m_e is the mass of electron, k is Boltzmann's constant, T_{M} is the matter temperature[5], h is Planck's constant, and χ_{HeII} is the ionization potential of He^{2+}.

For H and He I recombinations, the rate equations should be used. A common method for the calculation of the recombination process is sometimes called the Peebles recombination (Peebles 1968, 1993). This method considers a three-level atom with the ground state, first excited state and continuum. A recombination coefficient α is used in the calculation. If one excludes recombinations to the ground state and assumes the Lyman lines to be optically thick, the recombination is known as Case B recombination (Hu et al. 1995; Seager et al. 2000).

Here we consider hydrogen first. The treatment for helium is essentially the same. The ionization rate is given by

$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = C_{\mathrm{r}} \left[\beta_{\mathrm{H}}(T_{\mathrm{M}})(1-x_p)\mathrm{e}^{-h\nu_{\mathrm{H}\,2s}/kT_{\mathrm{M}}} - n_{\mathrm{H}}\alpha_{\mathrm{H}}(T_{\mathrm{M}})x_e x_p\right] \tag{5}$$

(Peebles 1968, 1993; Ma & Bertschinger 1995; Seager et al. 1999, 2000). Here C_{r} is a reduction factor (to be discussed below), $\nu_{\mathrm{H}\,2s}$ is the Lyα frequency, $\alpha_{\mathrm{H}}(T_{\mathrm{M}})$ is the Case B recombination rate for hydrogen, and the total photoionization rate $\beta_{\mathrm{H}}(T_{\mathrm{M}})$ is given by

[4] This form of the Saha equation is different from the usual one (e.g. in Ma & Bertschinger (1995)). Here we have followed the notation of Seager et al. (1999) and Seager et al. (2000). The left hand side is different because the ionization fractions are all defined relative to hydrogen.

[5] Matter is coupled by S-wave scattering (Hannestad 2001). By comparing the scattering rate and the Hubble expansion rate, it can be proved that the assumption of a single matter temperature is valid at least down to a redshift of several hundreds.

$$\beta_{\rm H} = \alpha_{\rm H} \left(\frac{2\pi m_e k T_{\rm M}}{h^2}\right)^{3/2} {\rm e}^{-h\nu_{{\rm H}\,2s}/kT_{\rm M}} \ . \tag{6}$$

Due to the large Lyman alpha and Lyman continuum opacities, recombination directly to the ground state will lead to immediate re-ionizaton (Peebles 1968; Ma & Bertschinger 1995). We therefore do not include recombination directly to the ground state. Recombination occurs either by 2s level to ground state decay (with rate $\Lambda_{\rm H}$), or by cosmological redshifting of Lyman alpha photons (with rate Λ_α). Since an atom in the 2s level may be ionized before decaying to ground state, the recombination rate is reduced. The reduction factor $C_{\rm r}$ is the ratio of the net decay rate to the sum of the decay and ionizaton rates from the 2s state, given by

$$C_{\rm r} = \frac{\Lambda_\alpha + \Lambda_{\rm H}}{\Lambda_\alpha + \Lambda_{\rm H} + \beta_{\rm H}(T_{\rm M})} \ , \tag{7}$$

where

$$\Lambda_\alpha = \frac{8\pi}{a\lambda_\alpha^3 n_{1s}} \frac{{\rm d}a}{{\rm d}t} \ , \ \lambda_\alpha = \frac{8\pi\hbar c}{3 \times 13.6 \ {\rm eV}} \ . \tag{8}$$

It is a very good approximation to replace n_{1s} by $(1 - x_{\rm H})n_{\rm H}$ for $T_{\rm M} \ll 10^5$ K.

Now all we need are the recombination coefficients for H and He II. The recombination coefficient of hydrogen $\alpha_{\rm H}$ is given by the fitting function (Seager et al. 2000)

$$\alpha_{\rm H} = 10^{-19} \frac{at^b}{1 + ct^d} \ {\rm m}^3 \, {\rm s}^{-1} \ , \tag{9}$$

with $a = 4.309$, $b = -0.6166$, $c = 0.6703$, $d = 0.5300$, and $t = T_{\rm M}/10^4$ K. The recombination coefficient of He I $\alpha_{\rm HeI}$ is fitted as[6]

$$\alpha_{\rm HeI} = q \left[\sqrt{\frac{T_{\rm M}}{T_2}} \left(1 + \sqrt{\frac{T_{\rm M}}{T_2}}\right)^{1-p} \left(1 + \sqrt{\frac{T_{\rm M}}{T_1}}\right)^{1+p} \right]^{-1} {\rm m}^3 \, {\rm s}^{-1} \ , \tag{10}$$

with $q = 10^{-16.744}$, $p = 0.711$, $T_1 = 10^{5.114}$ K and T_2 fixed arbitrarily at 3 K.

Note that in Seager et al. (1999), a fudge factor 1.14 is multiplied to Eq. 9 to approximate the result of a more detailed calculation. The detailed calculation includes totally 609 levels. There are 300 levels of H, 200 for He I, 100 for He II, 1 each for He III, electrons, protons, and each of the five molecular or ionic H species. The purpose of our work is not to reproduce the 609-level result. Since we just want to investigate the effect of taking into account the ionization energy of particles, and using the fudge factor will give unnecessary complication of the problem, we set the fudge factor to 1. Then the whole set of equations reduces to the Peebles recombination.

We can now write down the two ionization fraction rate equations for the proton fraction x_p and the singly ionized helium fraction $x_{\rm HeII} = n_{\rm HeII}/n_{\rm H}$, as described by (Seager et al. 1999)

$$\frac{{\rm d}x_p}{{\rm d}z} = \frac{\left[x_e x_p n_{\rm H} \alpha_{\rm H} - \beta_{\rm H}(1 - x_p){\rm e}^{-h\nu_{{\rm H}\,2s}/kT_{\rm M}}\right] \left[1 + K_{\rm H}\Lambda_{\rm H} n_{\rm H}(1 - x_p)\right]}{H(z)(1 + z) \left[1 + K_{\rm H}(\Lambda_{\rm H} + \beta_{\rm H}) n_{\rm H}(1 - x_p)\right]} \ , \tag{11}$$

and

$$\frac{dx_{\rm HeII}}{dz} = \frac{\left[x_{\rm HeII} x_e n_{\rm H} \alpha_{\rm HeI} - \beta_{\rm HeI}(f_{\rm He} - x_{\rm HeII}){\rm e}^{-h\nu_{{\rm HeI}\,2^1s}/kT_{\rm M}}\right] \left[1 + K_{\rm HeI}\Lambda_{\rm He} n_{\rm H}(f_{\rm He} - x_{\rm HeII}){\rm e}^{-h\nu_{{\rm HeI}\,2^1p\,2^1s}/kT_{\rm M}}\right]}{H(z)(1 + z) \left[1 + K_{\rm HeI}(\Lambda_{\rm He} + \beta_{\rm HeI}) n_{\rm H}(f_{\rm He} - x_{\rm HeII}){\rm e}^{-h\nu_{{\rm HeI}\,2^1p\,2^1s}/kT_{\rm M}}\right]} \ . \tag{12}$$

The electron fraction is $x_e = x_p + x_{\rm HeII}$. The H Lyα wavelength is $\lambda_{{\rm H}\,2p} = 121.5682$ nm. The H $2s - 1s$ frequency is $\nu_{{\rm H}\,2s}$, nearly equal to $c/\lambda_{{\rm H}\,2p}$. The He I $2^1p - 1^1s$ wavelength is $\lambda_{{\rm HeI}\,2^1p} = 58.4334$ nm. The He I $2^1s - 1^1s$ frequency is $\nu_{{\rm HeI}\,2^1s} = c/60.1404$ nm, and $\nu_{{\rm HeI}\,2^1p\,2^1s}$ is given by $\nu_{{\rm HeI}\,2^1p\,2^1s} = \nu_{{\rm HeI}\,2^1p} - \nu_{{\rm HeI}\,2^1s}$. The H $2s - 1s$ two-photon rate is $\Lambda_{\rm H} = 8.22458 \ {\rm s}^{-1}$, and the He I $2^1s - 1^1s$ two-photon rate is $\Lambda_{\rm He} = 51.3 \ {\rm s}^{-1}$. $K_{\rm H} \equiv \lambda_{{\rm H}\,2p}^3/[8\pi H(z)]$ is the cosmological redshifting of H Lyα photons, which is the reciprocal of Λ_α in Eq. 7. The cosmological redshifting of He I $2^1p - 1^1s$ photons is $K_{\rm HeI} \equiv \lambda_{{\rm HeI}\,2^1p}^3/[8\pi H(z)]$. The radiation temperature is given by $T_{\rm R} = T_0(1+z)$, with T_0 being the present CMB temperature.

2.3 Matter Temperature

The major effects affecting the evolution of matter temperature are Compton cooling and adiabatic cooling. Other cooling and heating terms have effects at 10^{-3}% level in the ionization fraction (Seager et al. 2000), and so we also ignore those terms in our calculation.

[6] There is a typo error in Seager et al. (1999) Eq. 4.

Electrons and photons are coupled by Compton scattering. When electrons and photons are nearly in thermal equilibrium, the rate of energy transfer is given by (Peebles 1968, 1993; Seager et al. 2000)

$$\frac{\mathrm{d}E_e}{\mathrm{d}t} = \frac{4\sigma_{\mathrm{T}} U n_e k}{m_e c}(T_{\mathrm{R}} - T_{\mathrm{M}}) \,, \tag{13}$$

where E_e is the energy density of electrons and σ_{T} is the Compton scattering cross section. The matter temperature therefore obeys:

$$\frac{\mathrm{d}T_{\mathrm{M}}}{\mathrm{d}t} = \frac{8}{3}\frac{\sigma_{\mathrm{T}} U}{m_e c}\frac{n_e}{n_{\mathrm{tot}}}(T_{\mathrm{R}} - T_{\mathrm{M}}) \,, \tag{14}$$

where n_{tot} is the total number density of particles.

We next consider the adiabatic cooling term. We treat the matter as an ideal gas with an adiabatic index $\gamma = 5/3$. Recalling that $T_{\mathrm{M}} \propto \rho^{\gamma-1}$ and $\rho \propto (1+z)^3$ (ignoring the pressure of baryonic matter in the Friedmann equation[7]), it can be proved easily that the adiabatic cooling rate is described by

$$\frac{\mathrm{d}T_{\mathrm{M}}}{\mathrm{d}t} = -3(\gamma-1)H(t)T_{\mathrm{M}} = -2H(t)T_{\mathrm{M}} \,. \tag{15}$$

Therefore the evolution of the matter temperature with respect to redshift z is

$$\frac{\mathrm{d}T_{\mathrm{M}}}{\mathrm{d}z} = \frac{8\sigma_{\mathrm{T}} U}{3H(z)(1+z)m_e c}\,\frac{n_e}{n_e + n_{\mathrm{H}} + n_{\mathrm{He}}}\,(T_{\mathrm{M}} - T_{\mathrm{R}}) + \frac{2T_{\mathrm{M}}}{(1+z)} \,. \tag{16}$$

By solving Eq. 11, Eq. 12 and Eq. 16, the ionization fractions and matter temperature can be found as a function of z.

3 RECOMBINATION SOFTENING

In the derivation of Eq. 16, we have assumed that the baryonic matter is an ideal gas. It is a good approximation when the gas is either neutral or fully ionized. However, a partially ionized gas tends to recombine under compression, and so the EOS should be softer than that of an ideal gas. Here we rewrite the temperature equation and investigate the effect.

Again we consider the recombination of H and He I only. The specific internal energy is the sum of the translation, excitation, and ionization energies of the particles H and He I per gram of materials:

$$e = \frac{1}{n_{\mathrm{H}}(m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}})}\left[\frac{3}{2}(n_{\mathrm{H}} + n_e + n_{\mathrm{He}})kT_{\mathrm{M}} + \sum_i n_{i\,\mathrm{H}}\,\epsilon_{i\,\mathrm{H}} + n_p\,\epsilon_{\mathrm{H}} + \sum_i n_{i\,\mathrm{HeI}}\,\epsilon_{i\,\mathrm{HeI}} + n_{\mathrm{HeII}}\,\epsilon_{\mathrm{HeI}}\right] \,. \tag{17}$$

Here we have assumed that all He III have already recombined to He II, and so the total number density of particles is $n_{\mathrm{H}} + n_e + n_{\mathrm{He}}$. $\epsilon_{i\,\mathrm{H}}$ is the excitation energy of the i^{th} excited state of H, ϵ_{H} is the ionization energy of H, $\epsilon_{i\,\mathrm{HeI}}$ is the excitation energy of the i^{th} excited state of He I, and ϵ_{HeI} is the ionization energy of He I.

Recalling the definition of $f_{\mathrm{He}} = n_{\mathrm{He}}/n_{\mathrm{H}}$, we rewrite Eq. 17 as

$$\begin{aligned} e \;=\; & \frac{3}{2}(1 + x_e + f_{\mathrm{He}})\frac{k}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}T_{\mathrm{M}} + \left[\frac{1-x_p}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\sum_i \frac{n_{i\,\mathrm{H}}}{(1-x_p)n_{\mathrm{H}}}\,\epsilon_{i\,\mathrm{H}} + \frac{x_p\,\epsilon_{\mathrm{H}}}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\right. \\ & \left. + \frac{f_{\mathrm{He}} - x_{i\,\mathrm{HeI}}}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\sum_i \frac{n_{i\,\mathrm{HeI}}}{(f_{\mathrm{He}} - x_{i\,\mathrm{HeI}})n_{\mathrm{H}}}\,\epsilon_{i\,\mathrm{HeI}} + \frac{x_{\mathrm{HeII}}\,\epsilon_{\mathrm{HeI}}}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\right] \,. \end{aligned} \tag{18}$$

The excitation energy terms in Eq. 18 are found to be negligible[8], and so we have

$$\begin{aligned} \mathrm{d}e \;=\; & \frac{3}{2}(1 + x_e + f_{\mathrm{He}})\frac{k}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\mathrm{d}T_{\mathrm{M}} + \left(\frac{3}{2}\frac{k}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}T_{\mathrm{M}} + \frac{\epsilon_{\mathrm{H}}}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\right)\mathrm{d}x_p \\ & + \left(\frac{3}{2}\frac{k}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}T_{\mathrm{M}} + \frac{\epsilon_{\mathrm{HeI}}}{m_{\mathrm{H}} + f_{\mathrm{He}} m_{\mathrm{He}}}\right)\mathrm{d}x_{\mathrm{HeII}} \,. \end{aligned} \tag{19}$$

On the other hand, we can also write for an adiabatic process

$$\mathrm{d}e = \left(\frac{p}{\rho^2}\right)\mathrm{d}\rho \,, \tag{20}$$

[7] By Friedmann equations, $\dot{\rho}c^2 a + 3(\rho c^2 + p)\dot{a} = 0$. Since the baryon pressure is much smaller than the energy density ρc^2, even though the pressure of baryon will be halved during the recombination process, it is still a good approximation to ignore the pressure.
[8] To be explained in the appendix.

where p is the pressure, given by the ideal gas law[9] as

$$p = (1 + f_{\rm He} + x_p + x_{\rm HeII})\,\rho\,\frac{k}{m_{\rm H} + f_{\rm He} m_{\rm He}}\,T_{\rm M}\,. \tag{21}$$

Now we may eliminate p to obtain an equation with ρ, $T_{\rm M}$, x_p, $x_{\rm HeII}$ only. Note that our method is similar to that in section 14 of Mihalas & Mihalas (1999), but there is an important difference. In Mihalas & Mihalas (1999), the Saha equation is used to eliminate the ionization fraction. We certainly should not use the Saha equation in this case, but instead we will keep the derivative of the ionization fraction. Putting Eq. 21 into Eq. 20 to eliminate p, we get

$$\mathrm{d}e = \left[(1 + f_{\rm He} + x_p + x_{\rm HeII})\,\frac{k}{m_{\rm H} + f_{\rm He} m_{\rm He}}\,T_{\rm M}\right]\frac{\mathrm{d}\rho}{\rho}\,, \tag{22}$$

which can then be combined with Eq. 19 and $\rho \propto (1+z)^3$ to obtain the modified temperature equation

$$\begin{aligned}\frac{1+z}{T_{\rm M}}\frac{\mathrm{d}T_{\rm M}}{\mathrm{d}z} \quad = \quad & \frac{8\sigma_{\rm T} U}{3H(z) m_e c}\,\frac{n_e}{n_e + n_{\rm H} + n_{\rm He}}\,\frac{T_{\rm M} - T_{\rm R}}{T_{\rm M}} + 2 - \left(1 + \frac{2}{3}\,\frac{\epsilon_{\rm H}}{kT_{\rm M}}\right)\frac{1+z}{1 + f_{\rm He} + x_e}\,\frac{\mathrm{d}x_p}{\mathrm{d}z} \\ & - \left(1 + \frac{2}{3}\,\frac{\epsilon_{\rm HeII}}{kT_{\rm M}}\right)\frac{1+z}{1 + f_{\rm He} + x_e}\,\frac{\mathrm{d}x_{\rm HeII}}{\mathrm{d}z}\,.\end{aligned} \tag{23}$$

Eq. 23 should then be used instead of the original equation Eq. 16. Hereafter we call the first term on the right to be the Compton Scattering term, the second the original adiabatic cooling term, the third and last the modified hydrogen and helium terms respectively.

4 RESULTS

Now we investigate the effects of recombination softening and reheating. The model that we consider is the ΛCDM model, with dark energy, CDM, baryon and radiation as the components in the universe. Our choice of the cosmological parameters bases on the Running Spectral Index Model of *WMAP* (Bennett et al. 2003; Spergel et al. 2003). In this model, the universe is flat, and so the total density parameter Ω_0 is 1.0 and $\Omega_{\rm K}$ is 0.0. The contribution of the dark energy Ω_Λ is 0.73, and the total matter density $\Omega_{\rm M}$ is 0.27, in which the CDM density $\Omega_{\rm C}$ is 0.226 and the baryon density $\Omega_{\rm B}$ is 0.044. The EOS index of dark energy is equal to -0.78 and is fixed. We use $h = 0.71$, $T_0 = 2.725$ K and $Y_{\rm p} = 0.24$. The scalar spectral index $n_{\rm s}$ is 0.93, and the derivative of spectral index $\mathrm{d}n_{\rm s}/\mathrm{d}\ln k$ is -0.031. We also simplify the calculation by assuming no reionization, lensing effect, tensor perturbations and massive neutrino in both models. Besides, we use *COBE* normalization throughout our calculation.

We have modified the code RECFAST in CMBFAST 4.3 (Seljak & Zaldarriaga 1996). The recombination process of He III to He II is calculated using the Saha equation. We solved a set of three differential equations, namely, the ionization equations for H and He I and the modified temperature equation for matter Eq. 23. The equations are then integrated by using the integrator DVERK (Seager et al. 1999). As in the original RECFAST, we turn off He II whenever its ionization fraction is small enough. We use a fudge factor 1 in our calculation. However, we find that the conclusion that we make below is unchanged if we use the fudge facter used in RECFAST.

Fig. 1 shows the values of the various terms on the r.h.s. of the modified temperature equation Eq. 23. The recombination reheating tends to delay the recombination process. However, Compton scattering keeps the matter temperature close to the radiation temperature, because the ionization fraction is still quite large. This explains the shapes of the curves in Fig. 1. This tight-coupling effect in fact has reduced the effect of recombination softening and reheating.

The sum of all four terms in Eq. 23 is plotted in the upper panel of Fig. 2. At large z, the coefficient is equal to 1. This can be understood by knowing that matter and photons couple very well before recombination. Because the heat capacity of radiation is much larger than that of matter (Peebles 1993), the matter temperature will follow that of photons. The value of the coefficient is thus equal to $3(\gamma - 1) = 3(4/3 - 1) = 1$. The coefficient deviates from 1 as z decreases. That is because the ionization fraction drops so fast that the Compton scattering can no longer force matter to follow the photon temperature. The percentage difference in $(1+z)\,\mathrm{d}(\ln T_{\rm M})/\mathrm{d}z$ with and without recombination softening and reheating effects is shown in the lower panel of Fig. 2. When the recombination starts, the sum of the terms is smaller than the original calculation; this means the temperature decreases more slowly after the modification.

The percentage difference in the ionization fraction with and without recombination softening and reheating is plotted in Fig. 3. The original ionization history is shown as a subpanel. As the redshift decreases, the ionization fraction x_e drops.

[9] Since the recombination time scale is long compared to that to establish thermal equilibrium in the cosmic plasma, the composition of the baryon gas remains nearly constant. Due to this fact and also the fact that kinetic energy of baryons during recombination is much smaller than the excitational energy of hydrogen, the ideal gas law is valid. However, the adiabatic index is changed because of the changing degrees of freedom during recombination.

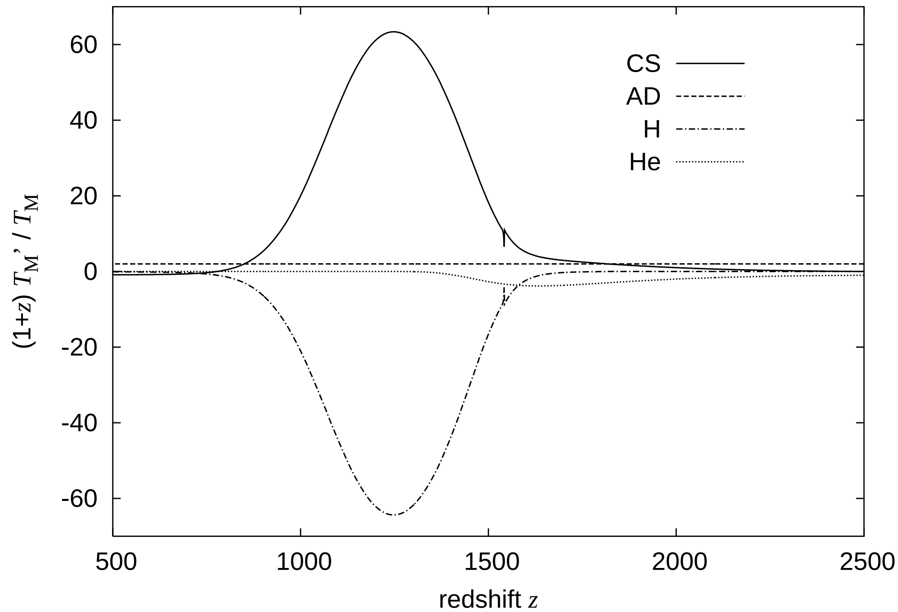

Figure 1. The z dependence of the various terms of the modified temperature equation Eq. 23. The solid, dashed, dot-dashed, and dotted lines are the Compton Scattering (CS), adiabatic cooling (AD), and the modified Hydrogen (H) and Helium (He) terms respectively.

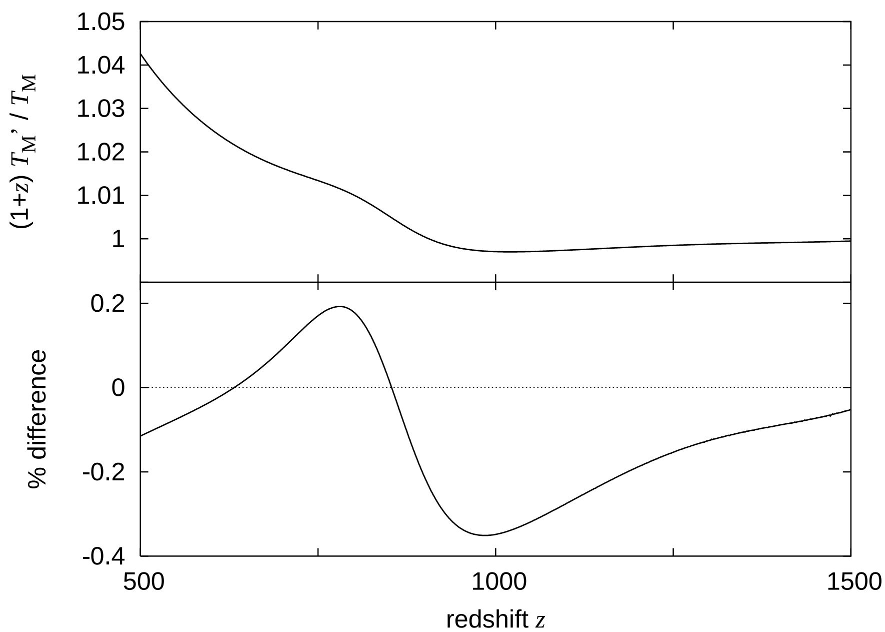

Figure 2. Sum of the terms on the r.h.s. of the temperature equation (upper panel) and the percentage difference with and without recombination softening and reheating effects (lower panel). A fudge factor of 1 has been used in the calculation.

The terms $\mathrm{d}x_p/\mathrm{d}z$ and $\mathrm{d}x_{\mathrm{HeII}}/\mathrm{d}z$ in the new temperature equation are not zero for decreasing ionization fraction, and so the ionization fraction is also modified.

With the ionization fraction at different redshift z, we can then calculate the visibility function $g(z) = \exp(-\tau)\mathrm{d}\tau/\mathrm{d}\eta(z)$, where τ is the optical depth, $\eta(z)$ is the conformal time. The peak of the visibility function defines the time of recombination, whereas its width defines the thickness of the last scattering surface. The visibility function and its changes are shown in Fig. 4, which clearly shows that the softening and reheating effect has delayed the recombination process.

We can understand the effect of the delay in recombination in terms of entropy. The specific entropy s (entropy per unit

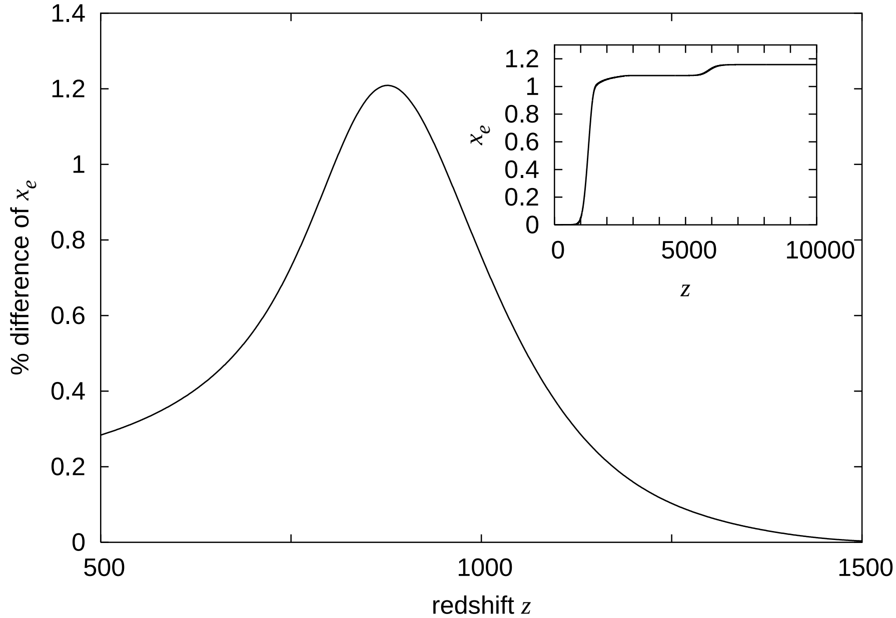

Figure 3. Change in the ionization fraction due to recombination softening and reheating. The subpanel shows the ionization history from between present and $z = 10000$.

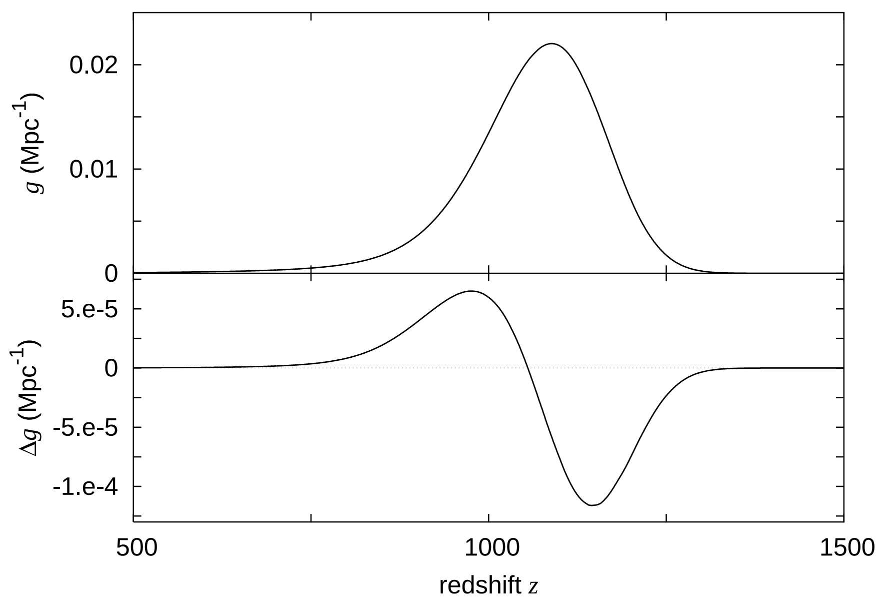

Figure 4. The visibility function g and its change Δg when recombination softening and reheating are inluded. A fudge factor of 1 has been used in the calculation.

mass) is defined by $T\,\mathrm{d}s = \mathrm{d}e + p\,\mathrm{d}(1/\rho)$. For matter, the change of specific entropy in the recombination process is

$$\begin{aligned}
\mathrm{d}s &= \frac{1}{T_\mathrm{M}}\left[\mathrm{d}e - \left(\frac{p}{\rho^2}\right)\mathrm{d}\rho\right] \\
&= \frac{k(1+x_p+x_\mathrm{HeII}+f_\mathrm{He})}{m_\mathrm{H}+f_\mathrm{He}m_\mathrm{He}}\left[\frac{3}{2}\frac{\mathrm{d}T_\mathrm{M}}{T_\mathrm{M}} - 3\,\frac{\mathrm{d}z}{1+z} + \left(\frac{3}{2}+\frac{\epsilon_\mathrm{H}}{kT_\mathrm{M}}\right)\frac{\mathrm{d}x_p}{1+x_p+x_\mathrm{HeII}+f_\mathrm{He}}\right. \\
&\quad \left. + \left(\frac{3}{2}+\frac{\epsilon_\mathrm{HeII}}{kT_\mathrm{M}}\right)\frac{\mathrm{d}x_\mathrm{HeII}}{1+x_p+x_\mathrm{HeII}+f_\mathrm{He}}\right]. \qquad (24)
\end{aligned}$$

A certain amount of entropy is released when the ions and electrons recombined. The terms $\mathrm{d}x_p$ and $\mathrm{d}x_\mathrm{HeII}$ correspond

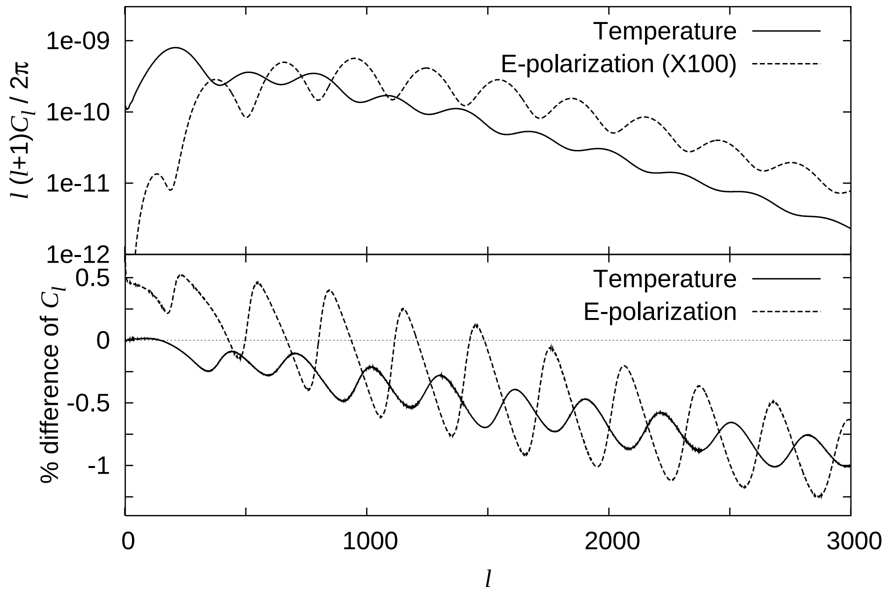

Figure 5. CMB temperature and E-polarization spectra (upper panel), and the percentage differences of the spectra C_l (lower panel), comparing results calculated with and without recombination softening and reheating effects.

to this effect. The entropy is then shared among the particles remaining in thermal contact, as reflected in the prefactor $1 + x_p + x_{\mathrm{HeII}} + f_{\mathrm{He}}$. As the gas recombines, the entropy is shared among less particles, thus the gas is heated[10] and the recombination process is delayed.

A delay of recombination changes the sound horizon at recombination, which directly affects the CMB power spectra. The new CMB power spectra and the percentage difference are shown in the lower panel of Fig. 5, with the original CMB spectra shown in the upper panel for comparison. Since the recombination is only delayed slightly, the change of peak positions is negligible. However, the change of amplitude of C_l can be as high as 1% to 2%. In the results of *WMAP*, the temperature fluctuations Δ_T at the first peak, the next trough, and the second peak are 74.7 ± 0.5 μK, 41.0 ± 0.5 μK, and 48.8 ± 0.9 μK respectively (Bennett et al. 2003). They are equivalent to uncertainties of 1.3%, 2.5% and 3.5% in C_l at $l = 220.1$, 411.7 and 546. Therefore, the error of calculation due to ignoring the recombination softening and reheating effect is about 10 times smaller than error of *WMAP*. However, *WMAP* is an ongoing project and the observational uncertainties will be reduced. Also, the next generation of CMB observations, such as *PLANCK*, may achieve precision better than 1% for $l < 1000$ (Bersanelli et al. 1996), and so the recombination softening and reheating effects should be included in future analysis.

Besides experimental error in C_l, there is a statistical error known as cosmic variance. The theoretical limit of precision in the determination of C_l is $\sim$1% at $l = 100$, and $\sim$0.1% at $l = 1000$ (Hu & Dodelson 2002). The effects of our modification are thus much larger than the cosmic variance for most l. Also, the effect of recombination softening and reheating increases as Ω_{B} increases. This is expected since our modification of the temperature equation is solely for the baryons. However, even when Ω_{B} is as large as 1, the change in C_l is only about twice as in the ΛCDM model. Thus our conclusion is the same in a large range of Ω_{B}.

The change of the CDM transfer function $T(k)$ is of order 0.01%. We normalize the transfer function at large scale to unity, so by definition there is no change in the large scale (small k). As the scale decreases, there is a region of $T(k)$ with signature of fluctuations in photon-baryon fluid. The delay of recombination will change the sound horizon and so will modify the fluctuations. At even smaller scales (large k), the fluctuations are damped by Silk damping. The damping length is about the geometric mean of the horizon size and the mean free path (Peacock 1999), and it is therefore larger in our calculation. This explains the decrease in the transfer function that we have found in our calculation. The new and old transfer functions approach a constant ratio because our calculation does not change the asymptotic behavior of the transfer function. The change in the transfer function is too small and can be safely neglected.

[10] One can also say that the cooling process is slowed down.

5 SUMMARY

We have presented all results using a fudge factor of 1 in the recombination calculation. We have also repeated the calculation using the fudge factor used in RECFAST, and the conclusion is essentially the same. Whether the fudge factor itself should be changed when taking recombination softening and reheating into account awaits further calculation.

The relative difference of the ionization fraction x_e calculated by the modified RECFAST with the new temperature equation and the original RECFAST increases as z decreases for $z < 300$. However, the temperature equation breaks down for small redshift. As the universe continues to cool down, complications such as the formation of molecules, formation of structure at different scales, and reionization dominate the physics.

In summary, we have studied the effects of recombination softening and reheating of the cosmic plasma on the ionization history, visibility function, the CMB spectra, and the CDM transfer function, using a generalized adiabatic index and standard cosmological parameters. With the standard cosmological parameters (ΛCDM model), the effects on the CMB spectra and ionization fraction are about 0.5% at $l = 1500$ and about 1% at $l = 3000$. These numbers are small compared to the current observational uncertainties but are not negligible. As the experimental errors become smaller, this improvement of the calculation should be used for determining the cosmological parameters. The effect on the CDM transfer function is much smaller, and so this modification is not important in the calculation of matter power spectrum.

The work described in this paper was substantially supported by a grant from the Research Grants Council of the Hong Kong Special Administrative Region, China (Project No. 400803), and a Direct Grant (Project Code: 2060248) from the Chinese University of Hong Kong.

REFERENCES

Bennett C. L., et al., 2003, ApJS, 148, 1
Spergel D. N., et al., 2003, ApJS, 148, 175
Bersanelli M., et al., Report on the Phase A Study of *COBRAS/SAMBA* (*PLANCK* project was originally named *COBRAS/SAMBA*)
Chandrasekhar S., 1939, An Introduction to the Study of Stellar Structure, Univ. of Chicago Press, Chicago
Drake G. W. F., 1996, Atomic, Molecular, & Optical Physics Handbook, AIP Press, Woodbury, NY, chapter 11
Durrer R., 2001, J. Phys. Stud., 5, 177
Hannestad S., 2001, New Astronomy, 6, 17
Hu W., Dodelson S., 2002, ARA&A, 40, 171
Hu W., Scott D., Sugiyama N., White M., 1995, ApJ, 52, 5498
Lang K. R., 1980, Astrophysical Formulae, 2nd edition, Springer-Verlag, Berlin, section 2.18
Ma C.-P., Bertschinger E., 1995, ApJ, 455, 7
Mihalas D., Mihalas B. W., 1999, Foundations of Radiation Hydrodynamics, Dover, Mineola, NY
Peacock J. A., 1999, Cosmological Physics, Cambridge Univ. Press, Cambridge, UK
Peebles P. J. E., 1968, ApJ, 153, 1
Peebles P. J. E., 1993, Principles of Physical Cosmology, Princeton Univ. Press, Princeton, NJ
Seager S., Sasselov D. D., Scott D., 1999, ApJ, 523, L1
Seager S., Sasselov D. D., Scott D., 2000, ApJS, 128, 407
Seljak U., Matias Z., 1996, ApJ, 469, 437

6 APPENDIX

6.1 Comparsion of the energy terms

For the density and temperature of the baryon gas at recombination, the major effect of line broadening is thermal broadening. The gas is nearly in thermal equilibrium and so follows Maxwell's distribution. The broadening is therefore about the width of the gaussian function. See Lang (1980) for a more detailed discussion of the effect.

The thermal broadening can be calculated by $\nu(2kT_{\mathrm{M}}/m_{\mathrm{H}}c^2)^{1/2}$, where ν is the photon frequency. Seager et al. (2000) has shown that at recombination the energy separation between the $n = 300$ level and the continuum of hydrogen is nearly equal to the energy width of the thermal broadening. The electrons in level higher than 300 will then be ionized thermally. Similarly we should include only 300 levels in our calculation of the excitation energy of hydrogen. For helium, we should follow Seager et al. (2000) to use 200 levels. Without this truncation of states, the sum in the excitation terms will diverge unphysically.

After the He II recombination, the specific internal energy of baryonic matter is

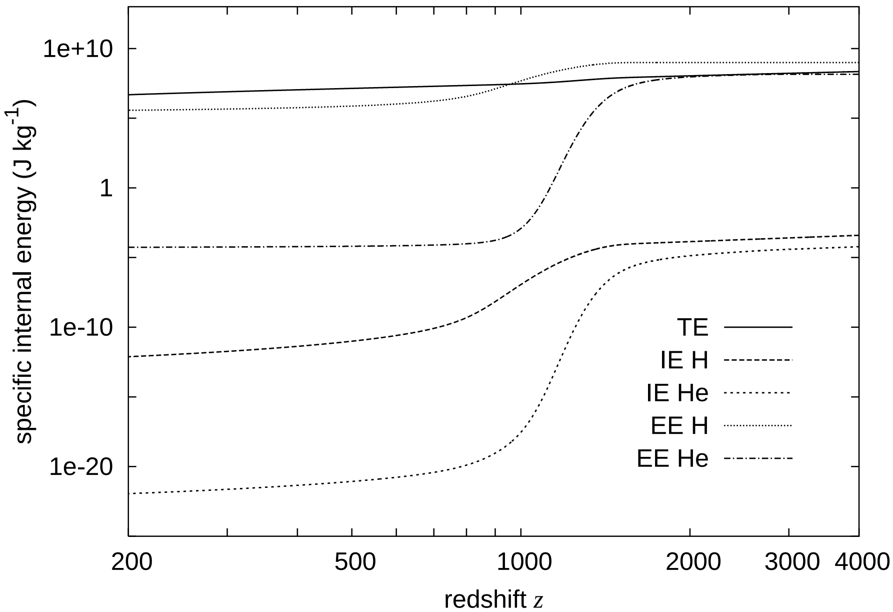

Figure 6. Different terms in the specific internal energy. The lines represent contribution from thermal energy of all particles (TE), ionization energy of hydrogen (IE H), ionization energy of helium I (IE He), excitation energy of hydrogen (EE H), and the upper limit of the excitation energy of helium I (EE He).

$$e = \frac{1}{n_{\rm H}\left(m_{\rm H} + f_{\rm He}m_{\rm He}\right)}\left[\frac{3}{2}n_{\rm H}(1+x_p+x_{\rm HeII}+f_{\rm He})kT_{\rm M} + \sum_i n_{i{\rm H}}\epsilon_{i{\rm H}} + n_p\epsilon_{\rm H} + \sum_i n_{i{\rm HeI}}\epsilon_{i{\rm HeI}} + n_{\rm HeII}\epsilon_{\rm HeI}\right], \tag{25}$$

where the first term is the thermal energy of all particles, the second and the third terms are the excitation energy and ionization energy of hydrogen atom respectively, and the fourth and fifth terms are the excitation energy and ionization energy of He I.

We are able to calculate the two ionization energy terms. We now simplify the hydrogen excitation energy term. When H starts to recombine, He has nearly recombined completely. The number density of H atoms with the electron in the $i^{\rm th}$ state is given by (Mihalas & Mihalas 1999)

$$n_{i{\rm H}} = 2Ci^2 n_e n_p T_{\rm M}^{-3/2} {\rm e}^{\epsilon_{\rm H}/i^2 kT_{\rm M}}, \tag{26}$$

with $C = (1/2)(h^2/2\pi m_e k)^{3/2}$.

The excitation energy in the $i^{\rm th}$ state of hydrogen is given by $\epsilon_{i{\rm H}} = (1-1/i^2)\epsilon_{\rm H}$, and so the excitation energy term of hydrogen is

$$\begin{aligned}\frac{1}{n_{\rm H}\left(m_{\rm H}+f_{\rm He}m_{\rm He}\right)}\sum_i n_{i{\rm H}}\epsilon_{i{\rm H}} &= \frac{1-x_p}{m_{\rm H}+f_{\rm He}m_{\rm He}}\sum_i\left(\frac{n_{i{\rm H}}}{n_{\rm H}-n_p}\right)\epsilon_{i{\rm H}} \\ &= \frac{1}{m_{\rm H}+f_{\rm He}m_{\rm He}}\,\frac{2C\rho_{\rm cr}\Omega_{\rm B}(1-Y_{\rm p})\epsilon_{\rm H}}{m_{\rm H}}x_p^2(1+z)^3 T_{\rm M}^{-3/2}\sum_i (i^2-1){\rm e}^{\epsilon_{\rm H}/i^2 kT_{\rm M}}, \end{aligned} \tag{27}$$

where $\rho_{\rm cr}$ is the critical density.

The excitation energy for He I is much more difficult to calculate. The helium atom is not hydrogenic. There is no close form for the energy states. One has to use methods like perturbation method or variational method. To calculate the energy states with high accuracy, one can refer to Drake (1996).

However for our purpose of checking the validity of neglecting the excitation terms, we do not have to calculate the correct energy states. Instead we find an upper limit of the states. We calculate the He I excitation energy with similar steps as the hydrogen. Then we construct a hydrogenic atom with a helium nucleus and one electron. Because we have neglected the repulsive force of another electron, the calculated energy in a certain state will be larger than the energy in the same state of the real helium atom. Therefore we can find a upper limit for the He I excitation energy.

Fig. 6 shows the five terms of the specific internal energy in Eq. 25. The two excitation energy terms are much smaller than the thermal energy of all particles and the ionization energy of hydrogen, and so it is appropriate to neglect the excitation energy terms. The He I ionization energy term is kept because it is important before and during the He I recombination.

This paper has been typeset from a TEX/ LATEX file prepared by the author.